MATH 147 Review: Arithmetic for Complex Numbers

Facts to Know

Complex Number is a number with two components, the real component and the imaginary component

Rectangular Form

$z = a + bi$ Here $i = \sqrt{-1}$

Polar Form

$z = r \cdot e^{i\theta}$

Addition

$z + w = (a + bi) + (c + di) =$ $(a+c) + (b+d)i$

Multiplication

$z \cdot w = (a + bi)(c + di) =$ $(ac - bd) + (bc + ad)i$

$z \cdot w = (r_1 e^{i\theta_1})(r_2 e^{i\theta_2}) =$ $(r_1 r_2)\, e^{i(\theta_1 + \theta_2)}$

Multiplicative Inverse

$\frac{1}{z} = \frac{1}{(a+bi)} \cdot \frac{a-bi}{a-bi} = \frac{a - bi}{a^2 + b^2}$

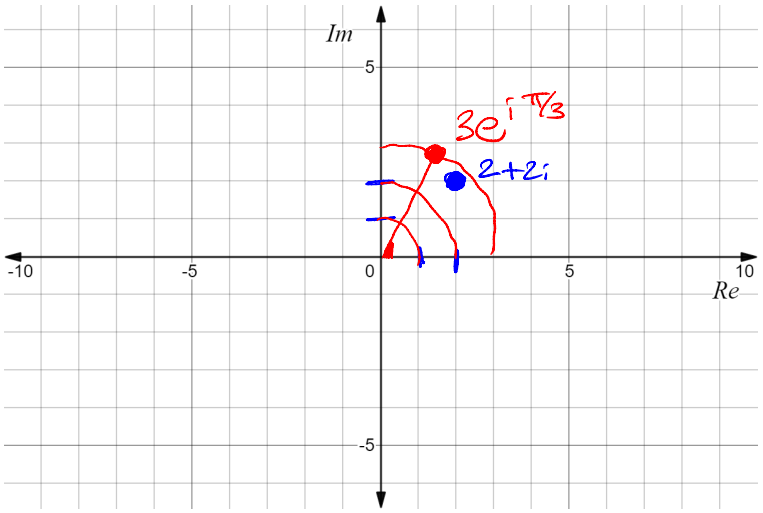
$\frac{1}{z} = \frac{1}{re^{i\theta}} =$

$= \left(\frac{1}{r}\right) e^{i(-\theta)}$

Examples

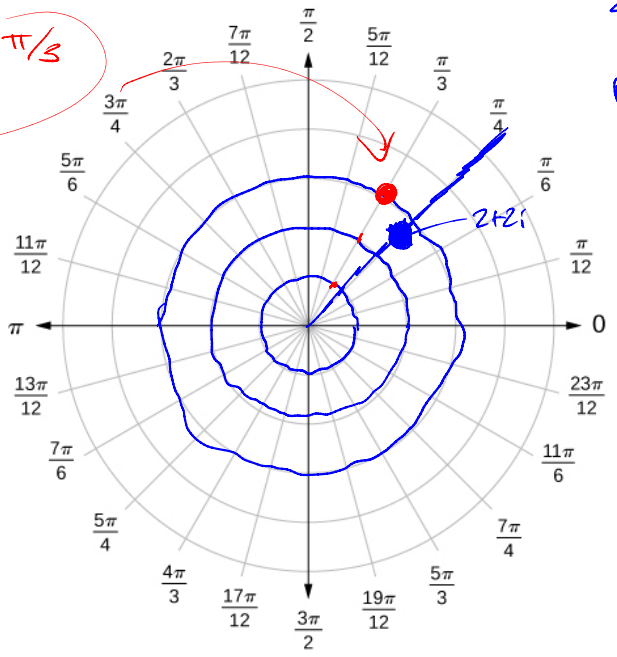
$\frac{\pi}{3} = 60°$

1. Plot $2 + 2i$ and $3e^{i\frac{\pi}{3}}$.



2. Plot $2 + 2i$ and $3e^{i\frac{\pi}{3}}$.

$z = 3e^{i\pi/3}$



$2+2i$

$r = \sqrt{2^2+2^2}$

$= \sqrt{8}$

$= 2\sqrt{2}$

$\approx 2(1.414)$

≈ 2.828

$\theta = 45°$

$= \frac{\pi}{4}$

3. Find $z + w$, $z \cdot w$, $\frac{1}{z}$ where $z = 1 - i$ and $w = 1 - \sqrt{3}i$.

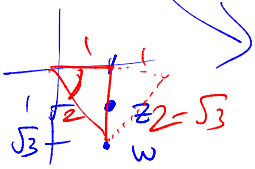
$$z + w = (1 + 1) + ((-1) + (-\sqrt{3}))\,i$$

$$= 2 + (-1 - \sqrt{3})\,i$$

$$z \cdot w = (1 - i)(1 - \sqrt{3}\,i)$$

$$= (1 - \sqrt{3}) + (-1 + (-\sqrt{3}))\,i$$

$$= (1 - \sqrt{3}) + (-1 - \sqrt{3})\,i$$



$$= \sqrt{2}\, e^{i\left(-\frac{\pi}{4}\right)} \cdot 2\, e^{i\left(-\pi/3\right)}$$

$$= 2\sqrt{2}\, e^{i\left(\frac{-7\pi}{12}\right)}$$

$$\frac{1}{z} = \frac{1 + i}{1^2 + 1^2} \qquad \overline{z} = 1 + i$$

$$\frac{1}{z} = \frac{1}{\sqrt{2}\, e^{i\frac{\pi}{4}}} = \left(\frac{1}{\sqrt{2}}\right) e^{i\left(+\frac{\pi}{4}\right)}$$